

October 2009

IPMU09-0121

# Number and Entropy of Halo Black Holes

Paul H. Frampton<sup>\*</sup> <sup>(a,b)</sup> and Kevin Ludwick<sup>†(a)</sup>

<sup>(a)</sup> *Department of Physics and Astronomy, University of North Carolina,  
Chapel Hill, NC 27599-3255.*

<sup>(b)</sup> *Institute for the Physics and Mathematics of the Universe, University of  
Tokyo, Kashiwa, Chiba 277-8568, Japan.*

## Abstract

Based on constraints from microlensing and disk stability, both with and without limitations from wide binary surveys, we estimate the total number and entropy of intermediate mass black holes. Given the visible universe comprises  $10^{11}$  halos each of mass  $\sim 10^{12}M_{\odot}$ , typical core black holes of mean mass  $\sim 10^7M_{\odot}$  set the dimensionless entropy ( $S/k$ ) of the universe at a thousand googols. Identification of all dark matter as black holes sets the dimensionless entropy of the universe at ten million googols, implying that dark matter can contribute over 99% of entropy, which favors all dark matter as black holes in the mass regime of  $\sim 10^5M_{\odot}$ .

---

<sup>\*</sup>frampton@physics.unc.edu

<sup>†</sup>kludwick@physics.unc.edu

## *Introduction*

The identification of dark matter, for which there is compelling evidence from its gravitational effects in galaxies and clusters thereof, is an important outstanding question. Dark matter makes up some eighty percent of matter and a quarter of the energy content of the universe. If dark matter is composed of only one constituent, its mass is uncertain by almost eighty orders of magnitude from  $10^{-15}$  to  $10^{+65}$  GeV.

Of course, it would be reassuring to identify dark matter by its production in particle colliders and by its detection in terrestrial experiments. On the other hand, the dark matter constituent may equally be, as assumed here, in a completely different and collider-inaccessible mass regime heavier than the Sun.

The observational limits on the occurrence of such multi-solar mass astrophysical objects in the halo have considerably changed recently. There remain microlensing limits [1, 2] on masses below a solar mass and slightly above. There are also respected limits from numerical study [3] of disk stability at ten million solar masses and slightly below.

For the intermediate mass region, a possible constraint comes from the occurrence of gravitationally bound binary stars at high separation approaching one parsec. Here the situation has changed recently, and the bounds are far more relaxed, possibly non-existent. The first such analysis [4] allowed only some ten percent of halo dark matter for most of the mass range. A more recent analysis [5] permits fifty percent and cautions that the sample of binaries may be too small to draw any solid conclusions.

In the following, we adopt constraints from microlensing and disk stability but keep an open mind with respect to the wide binaries. We estimate the total number and total entropy of the black holes per halo and hence (simply multiplying by  $10^{11}$ ) in the universe, assuming as in [6] that all dark matter can be identified as black holes.

## *Number of Black Holes*

To estimate number and subsequently entropy of black holes we simplify by taking as possible masses  $10^n M_\odot$  with  $n$  integer,  $1 \leq n \leq 7$ . Further, we assume the constraints from wide binaries [5] for different  $n$  are independent of each other.

We make our analysis first with binary constraints, denoted simply as “with”, then with no binary constraints, denoted as “without”. Let  $f_n$  be the fraction

of the halo dark matter composed of mass  $10^n M_\odot$  black holes. The total halo mass is taken to be  $10^{12} M_\odot$  whereupon

$$\Sigma_n f_n = 1 \quad (1)$$

and the number  $N_n$  is

$$N_n = f_n 10^{12-n}. \quad (2)$$

The “with” constraints on the  $f_n$  are

$$\begin{aligned} 0 &\leq f_1 \leq 0.4 \\ 0 &\leq f_2 \leq 1.0 \\ 0 &\leq f_3 \leq 0.5 \\ 0 &\leq f_{4,5,6} \leq 0.4 \\ 0 &\leq f_7 \leq 0.3. \end{aligned} \quad (3)$$

For the “without” constraints, the  $f_{1,2,7}$  ranges remain unchanged while the  $f_{3,4,5,6}$  are free, namely

$$\begin{aligned} 0 &\leq f_1 \leq 0.4 \\ 0 &\leq f_{2,3,4,5,6} \leq 1.0 \\ 0 &\leq f_7 \leq 0.3. \end{aligned} \quad (4)$$

Allowing the  $f_n$  to vary by increments  $\Delta f_n = 0.1$  for  $1 \leq f_n \leq (f_n)_{max}$  we allow the black holes to have  $\nu$  different mass (or  $n$ ) values with  $1 \leq \nu \leq 7$ . For the “with” constraints, we then find numbers of black holes as follows:

**Table 1**

$\nu$	# choices	$N_{mean}$	$N_{median}$	$N_{max}$	$N_{min}$
1	1	$1.0 \times 10^{10}$	$1.0 \times 10^{10}$	$1.0 \times 10^{10}$	$1.0 \times 10^{10}$
2	24	$1.2 \times 10^{10}$	$8.0 \times 10^9$	$4.6 \times 10^{10}$	$5.5 \times 10^9$
3	365	$1.4 \times 10^{10}$	$6.1 \times 10^9$	$4.5 \times 10^{10}$	$3.4 \times 10^6$
4	1660	$1.6 \times 10^{10}$	$1.2 \times 10^{10}$	$4.4 \times 10^{10}$	$1.2 \times 10^7$
5	2106	$1.6 \times 10^{10}$	$1.3 \times 10^{10}$	$4.3 \times 10^{10}$	$1.1 \times 10^8$
6	822	$1.6 \times 10^{10}$	$1.2 \times 10^{10}$	$4.2 \times 10^{10}$	$1.1 \times 10^9$
7	83	$1.6 \times 10^{10}$	$1.2 \times 10^{10}$	$4.1 \times 10^{10}$	$1.1 \times 10^{10}$

For the “without” case we find:

**Table 2**

$\nu$	# choices	$N_{mean}$	$N_{median}$	$N_{max}$	$N_{min}$
1	5	$2.2 \times 10^9$	$1.0 \times 10^9$	$1.0 \times 10^{10}$	$1.0 \times 10^6$
2	125	$6.1 \times 10^9$	$9.0 \times 10^8$	$4.6 \times 10^{10}$	$7.3 \times 10^5$
3	890	$1.0 \times 10^{10}$	$4.0 \times 10^9$	$4.5 \times 10^{10}$	$1.6 \times 10^6$
4	2340	$1.4 \times 10^{10}$	$1.0 \times 10^{10}$	$4.4 \times 10^{10}$	$1.1 \times 10^7$
5	2346	$1.5 \times 10^{10}$	$1.2 \times 10^{10}$	$4.3 \times 10^{10}$	$1.1 \times 10^8$
6	840	$1.6 \times 10^{10}$	$1.2 \times 10^{10}$	$4.2 \times 10^{10}$	$1.1 \times 10^9$
7	83	$1.6 \times 10^{10}$	$1.2 \times 10^{10}$	$4.1 \times 10^{10}$	$1.1 \times 10^{10}$

Study of Tables 1 and 2 reveals a number of things about the putative intermediate mass black holes which may dominate the matter content. First, the comparison of the tables reveals that the wide binary constraints, as they stand, do not affect the numbers very much. Thus, unless and until a much bigger sample of wide binaries is found (if they exist), the conclusions about numbers of black holes in a halo is insensitive to their consideration.

As expected from the defining formula, Eq. (2), the number of black holes per halo can range from about a million to a few times ten billion. By sampling distributions of the masses, not just a single mass, Tables 1 and 2 reveal that the most likely number is at the high end, close to ten billion per halo.

Since there are generically  $10^{11}$  halos, this implies a total number of about a billion trillion black holes in the universe.

### *Entropy of Black Holes*

We can similarly estimate the total entropy of the halo black holes by exploiting the BPH entropy formula [7–9], which says that for a black hole with mass  $M_{BH} = \eta M_\odot$ , the entropy is  $S_{BH} = 10^{78} \eta^2$ .

For the “with” case, this gives the numbers for halo entropy

**Table 3**

$\nu$	# choices	$S_{mean}$	$S_{median}$	$S_{max}$	$S_{min}$
1	1	$1.0 \times 10^{92}$	$1.0 \times 10^{92}$	$1.0 \times 10^{92}$	$1.0 \times 10^{92}$
2	24	$3.0 \times 10^{95}$	$7.1 \times 10^{93}$	$3.0 \times 10^{96}$	$6.4 \times 10^{91}$
3	365	$7.5 \times 10^{95}$	$2.2 \times 10^{95}$	$3.4 \times 10^{96}$	$1.5 \times 10^{92}$
4	1660	$1.1 \times 10^{96}$	$1.0 \times 10^{96}$	$3.4 \times 10^{96}$	$1.1 \times 10^{93}$
5	2106	$1.4 \times 10^{96}$	$1.2 \times 10^{96}$	$3.4 \times 10^{96}$	$1.1 \times 10^{94}$
6	822	$1.5 \times 10^{96}$	$1.2 \times 10^{96}$	$3.3 \times 10^{96}$	$1.1 \times 10^{95}$
7	83	$1.6 \times 10^{96}$	$1.2 \times 10^{96}$	$3.2 \times 10^{96}$	$1.1 \times 10^{96}$

while for the “without” case we find

**Table 4**

$\nu$	# choices	$S_{mean}$	$S_{median}$	$S_{max}$	$S_{min}$
1	5	$2.2 \times 10^{95}$	$1.0 \times 10^{95}$	$1.0 \times 10^{96}$	$1.0 \times 10^{92}$
2	125	$4.5 \times 10^{95}$	$8.0 \times 10^{94}$	$3.7 \times 10^{96}$	$6.4 \times 10^{91}$
3	890	$7.7 \times 10^{95}$	$3.0 \times 10^{95}$	$3.6 \times 10^{96}$	$1.5 \times 10^{92}$
4	2340	$1.1 \times 10^{96}$	$1.0 \times 10^{96}$	$3.5 \times 10^{96}$	$1.1 \times 10^{93}$
5	2346	$1.3 \times 10^{96}$	$1.2 \times 10^{96}$	$3.4 \times 10^{96}$	$1.1 \times 10^{94}$
6	840	$1.5 \times 10^{96}$	$1.2 \times 10^{96}$	$3.3 \times 10^{96}$	$1.1 \times 10^{95}$
7	83	$1.6 \times 10^{96}$	$1.2 \times 10^{96}$	$3.2 \times 10^{96}$	$1.1 \times 10^{96}$

Tables 3 and 4 contain much information germane to the central idea that dark matter be identified as black holes.

The biggest known contributor of black holes in a halo is the core supermassive black hole (SMBH). In the Milky Way it is Sag A\* and for a typical galaxy a core SMBH has mass  $M_{SMBH} \sim 10^7 M_\odot$ . Its PBH entropy is therefore about  $\sim 10^{92}$ .

Multiplying by  $10^{11}$ , the number of halos, shows that these SMBHs contribute about  $10^{103}$ , or a thousand googols, to the entropy of the universe as emphasized in [6].

The conventional wisdom is that the SMBHs are the single dominant contributor to the entropy of the universe, which is therefore about a thousand googols.

From our Tables 3 and 4 we can arrive at a very different conclusion.

## *Reconsideration of the Entropy of the Universe*

Let us take the viewpoint that the universe, by which we mean the visible universe, is an isolated system in the usual sense of thermodynamics and statistical mechanics. In accord with the usual statistical law of thermodynamics, the entropy of the universe will increase to its maximum attainable value.

The natural unit for the dimensionless entropy of the universe  $S/k = \ln \Omega$  is the googol ( $10^{100}$ ). The supermassive black holes (SMBHs) at galactic cores contribute about a thousand googols.

The holographic bound [10] on information or entropy contained in a three-volume is that it be not above the surface area as measured in Planck units ( $10^{-33}cm^2$ ). If we take the visible universe to be a sphere of radius  $3 \times 10^{10}ly \sim 3 \times 10^{18}cm$ , the maximum entropy is  $\sim 10^{124}$  or a trillion trillion googols. This would be the entropy if the universe were one black hole of mass  $10^{23}M_{\odot}$ .

The numbers in Tables 3 and 4 suggest that the entropy contribution from dark matter can exceed that of the SMBHs by orders of magnitude. Taking the view that increasing total entropy plays a dominant role in cosmological evolution strongly favors the formation of black holes in the  $10^5M_{\odot}$  mass range and the view that they constitute all dark matter.

We are more confident about the present status of dark matter than of its detailed history but, of course, an interesting and legitimate question is: how did the black holes originate? One possible formation is as remnants of Population-III (henceforth Pop-III) stars formed at a redshift  $Z \sim 25$ . These Pop-III stars are necessary to explain the metallicity of Pop-I and Pop-II stars that formed later. Such Pop-III stars are not well understood but we expect they can be very massive,  $10^5M_{\odot}$ , to live for a short time, less than a million years, then explode leaving black holes which have a total mass that is a significant fraction of the original star's mass. Nevertheless, it is very unlikely [11] that a sufficient number of Pop-III stars can form to make all dark matter. Thus, the IMBHs may have formed in the early universe as primordial black holes.<sup>#1</sup>

---

<sup>#1</sup>Note that the constraints in [12] apply at the recombination era and subsequent black hole mergers can occur.

The first item of business is therefore to confirm that there are millions of large black holes in our halo and in others.

The ESA Gaia project is planned to survey billions of stars in our galaxy, the Milky Way, and should enable obtaining a large sample of gravitationally bound wide binaries which can be analyzed for evidence of black holes perturbing them.

The goal of the SuperMACHO project is to identify the objects which produced existing microlensing events and should allow the observation of higher longevity microlensing signals corresponding to the mass ranges suggested for the dark matter black holes.

Finally, if we truncate to  $n \leq 5$ , since Pop-III stars or IMBHs with higher masses seem unlikely [11], then the typical number of IMBHs per halo is  $\sim 10^{10}$ , giving about one million googols for the entropy of the universe. The majority of entropy may be concentrated in a tiny fraction of the total number of black holes as can be seen by studying examples, *e.g.*,  $f_2 = f_5 = 0.5$ .

The key motivation for our believing this interpretation of dark matter, as opposed to an interpretation involving microscopic particles, comes from consideration of the entropy of the universe. The SMBHs at galactic cores contribute about a thousand googols to the overall dimensionless entropy. As seen in the present article, dark matter in the form of black holes can contribute as much as a million googols and thus make up over 99% of the cosmic entropy, which is sufficient reason, if we adopt that the universe is an isolated system to which the second law of thermodynamics is applicable, for taking it seriously.

Hopefully future observations will be able to identify dark matter as black holes.

## Acknowledgements

This work was supported in part by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan and by the U.S. Department of Energy under Grant No. DE-FG02-06ER41418.



## References

- [1] C. Alcock *et al.* *Astrophys. J.* **486**, 697 (1997).
- [2] C. Alcock, *et al.* *Astrophys. J.* **542**, 281 (2000).
- [3] G.H. Xu and J.P. Ostriker, *Astrophys. J.* **437**, 184 (1994).
- [4] J. Yoo, J. Chaname, and A. Gould, *Astrophys.J.* **601**, 311 (2004).
- [5] D.P. Quinn. *et al.* [arXiv:0903.1644\[astro-ph\]](#).
- [6] P.H. Frampton, JCAP (in press). [arXiv:0905.3632\[hep-th\]](#)
- [7] L. Parker, *Phys. Rev.* **183**, 1057 (1969).
- [8] J.D. Bekenstein, *Phys. Rev.* **D7**, 2333 (1973).
- [9] S.W. Hawking, *Commun. Math. Phys.* **43**, 1974 (1975).
- [10] G. 't Hooft, In *Salamfestschrift*, editors: A. Ali, J. Ellis and S. Randjbar-Daemi, World Scientific (1994), p. 284. [arXiv:gr-qc/9310026](#).
- [11] P. Madau, private communication (2009).
- [12] M. Ricotti, J. P. Ostriker and K.J. Mack, [arXiv:0709.0524\[astro-ph\]](#).